Subject Code: 24BM11RC02 **R-24** **Reg No:** | | | | | | | | | | | |

GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN

(AUTONOMOUS)

(Affiliated to Andhra University, Visakhapatnam)

I B.Tech. II Semester Regular Examinations, June/July – 2025

LINEAR ALGEBRA AND VECTOR CALCULUS

(Common to All branches)

1. All questions carry equal marks
2. Must answer all parts of the question at one place

Time: 3Hrs. **Max Marks: 70**

SCHEME OF EVALUATION

Q.No.	Sub Q.No.	Solution	Marks
1	a	Find the rank of the matrix $A=\begin{bmatrix}1&2&3&0\\2&4&3&2\\3&2&1&3\\6&8&7&5\end{bmatrix}$ by reducing it to echelon form. **Sol**: $A=\begin{bmatrix}1&2&3&0\\2&4&3&2\\3&2&1&3\\6&8&7&5\end{bmatrix}\begin{matrix}R_2:R_2-2R_1\\R_3:R_3-3R_1\\R_4:R_4-6R_1\end{matrix}\sim\begin{bmatrix}1&2&3&0\\0&0&-3&2\\0&-4&-8&3\\0&-4&-11&5\end{bmatrix}R_2\leftrightarrow R_4\sim\begin{bmatrix}1&2&3&0\\0&-4&-11&5\\0&-4&-8&3\\0&0&-3&2\end{bmatrix}$ $R_3:R_3-R_2\sim\begin{bmatrix}1&2&3&0\\0&-4&-11&5\\0&0&3&-2\\0&0&-3&2\end{bmatrix}R_4:R_4+R_3\sim\begin{bmatrix}1&2&3&0\\0&-4&-11&5\\0&0&3&-2\\0&0&0&0\end{bmatrix}$ Rank of A= $\rho(A)=$Number of non zero rows in its echelon form=3.	3M 2M 2M
	b	Find the values of λ and μ so that the equations $2x+3y+5z=9$, $7x+3y-2z=8$, $2x+3y+\lambda z=\mu$. **Sol**: $[A\|B]=\begin{bmatrix}2&3&5&9\\7&3&-2&8\\2&3&\lambda&\mu\end{bmatrix}$ $R_2:2R_2-7R_1, R_3:R_3-R_1$ $\sim\begin{bmatrix}2&3&5&9\\0&-15&-39&47\\0&0&\lambda-5&\mu-9\end{bmatrix}$ A non-homogeneous system $AX=B$ is consistent and will have infinite number of solutions if $\rho(A)=\rho(A\|B)=r<n=$number of unknowns. $If\lambda=5$ and $\mu=9$, then $\rho(A)=\rho(A\|B)=2<3$. $\therefore\lambda=5\,\&\,\mu=9$.	3M 2M 2M

2	a	Apply Gauss Elimination Method to solve the equations $x+4y-z=-5, x+y-6z=12, 3x-y-z=4.$ **Sol**: $[A\|B]=\begin{bmatrix}1 & 4 & -1 & -5\\ 1 & 1 & -6 & 12\\ 3 & -1 & -1 & 4\end{bmatrix}$ $\begin{matrix}R_2: R_2-R_1\\ R_3: R_3-3R_1\end{matrix} \sim \begin{bmatrix}1 & 4 & -1 & -5\\ 0 & -3 & -5 & 17\\ 0 & -13 & 2 & 19\end{bmatrix}$ $R_3: 3R_3-13R_2 \sim \begin{bmatrix}1 & 4 & -1 & -5\\ 0 & -3 & -5 & 17\\ 0 & 0 & 71 & -164\end{bmatrix}$ $\Rightarrow x+4y-z=-5, -3y-5z=17, 71z=-164$ By back substitution, $z=-2.3098$ $y=-1.817$ $x=-0.0418$	4M 1M 2M
	b	Apply factorization method to solve the equations $3x+2y+7z=4, 2x+3y+z=5, 3x+4y+z=7.$ **Sol**: $A=\begin{bmatrix}3 & 2 & 7\\ 2 & 3 & 1\\ 3 & 4 & 1\end{bmatrix}$ $A=LU=\begin{bmatrix}1 & 0 & 0\\ l_{21} & 1 & 0\\ l_{31} & l_{32} & 1\end{bmatrix}\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 2/3 & 1 & 0\\ 1 & 6/5 & 1\end{bmatrix}\begin{bmatrix}3 & 2 & 7\\ 0 & 5/3 & -11/3\\ 0 & 0 & 8/5\end{bmatrix}$ Now the system can be written as $LUX=B$ Letting $UX=V$ we get $LV=B$ $LV=B\Rightarrow\begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix}=\begin{bmatrix}4\\ 7/3\\ 1/5\end{bmatrix}$ (By forward substitution) $UX=V\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-1/8\\ 9/8\\ 7/8\end{bmatrix}$ (By back substitution)	3M 2M 2M
3	a	Verify Cayley-Hamilton Theorem for the matrix $A=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$ and find its inverse. **Sol**: Cayley-Hamilton theorem statement Charecteristic equation of A is $\lambda^2-7\lambda-2=0$ $A^2=\begin{bmatrix}16 & 21\\ 28 & 37\end{bmatrix}$ Proving $A^2-7A-2I=0$ $A^{-1}=\frac{1}{2}[A-7I]=\frac{1}{2}\begin{bmatrix}-5 & 3\\ 4 & -2\end{bmatrix}$	1M 2M 2M 2M

	b	Find the singular value decomposition of $A=\begin{bmatrix}1&1\\1&0\\0&1\end{bmatrix}$. **Sol**: $A^T A=\begin{bmatrix}2&1\\1&2\end{bmatrix}$ Characteristic equation of $A^T A$ is $\lambda^2-4\lambda+3=0$. $\Rightarrow \lambda=1,3.$ $\therefore$ Singular Values of A are $\sigma_1=\sqrt{3},\sigma_2=1$. Eigen vectors corresponding to 3,1 are $\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}1\\-1\end{bmatrix}$ respectively. $\therefore V_{2\times2}=[v_1\quad v_2]=\begin{bmatrix}1/\sqrt{2}&1/\sqrt{2}\\1/\sqrt{2}&-1/\sqrt{2}\end{bmatrix}$ $\&\sum_{3\times2}=\begin{bmatrix}\sqrt{3}&0\\0&1\\0&0\end{bmatrix}$ Now $U=[u_1\quad u_2\quad u_3]$ where $u_i=\frac{1}{\sigma_i}Av_i$ for $i=1,2,3$. $u_1=\begin{bmatrix}2/\sqrt{6}\\1/\sqrt{6}\\1/\sqrt{6}\end{bmatrix}\&u_2=\begin{bmatrix}0\\1/\sqrt{2}\\-1/\sqrt{2}\end{bmatrix}$ Let $u_3=\begin{bmatrix}a\\b\\c\end{bmatrix}$ be orthogonal to $u_1\&u_2$. $\Rightarrow u_3=\alpha\begin{bmatrix}-1\\1\\1\end{bmatrix}$. Normalized vector $u_3=\begin{bmatrix}-1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{bmatrix}$. $\therefore U=\begin{bmatrix}2/\sqrt{6}&0&-1/\sqrt{3}\\1/\sqrt{6}&1/\sqrt{2}&1/\sqrt{3}\\1/\sqrt{6}&-1/\sqrt{2}&1/\sqrt{3}\end{bmatrix}$ Thus $A=U\Sigma V^T=\begin{bmatrix}2/\sqrt{6}&0&-1/\sqrt{3}\\1/\sqrt{6}&1/\sqrt{2}&1/\sqrt{3}\\1/\sqrt{6}&-1/\sqrt{2}&1/\sqrt{3}\end{bmatrix}\begin{bmatrix}\sqrt{3}&0\\0&1\\0&0\end{bmatrix}\begin{bmatrix}1/\sqrt{2}&1/\sqrt{2}\\1/\sqrt{2}&-1/\sqrt{2}\end{bmatrix}$	2M 1M 1M 2M 1M
4		Find the eigen values and eigen vectors of the matrix A and A^{-1} where $A=\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$. **Sol**:Characteristic equation of A is $\lambda^3-7\lambda^2+36=0$. Eigen values of A are -2,3,6. Eigen vectors are $\begin{bmatrix}-1\\0\\1\end{bmatrix},\begin{bmatrix}1\\-1\\1\end{bmatrix},\begin{bmatrix}1\\2\\1\end{bmatrix}$.	3M 2M 6M

		RESULT: If λ is en eigen value of A then $\frac{1}{\lambda}$ is an eigen value of A^{-1}. Therefore the eigen values of A^{-1} are $\frac{-1}{2},\frac{1}{3},\frac{1}{6}$ and the corresponding eigen vectors are $\begin{bmatrix}-1\\0\\1\end{bmatrix},\begin{bmatrix}1\\-1\\1\end{bmatrix},\begin{bmatrix}1\\2\\1\end{bmatrix}$.	2M 1M
5		Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ into the canonical form by an orthogonal reduction and find its nature. **Sol**: Matrix of the quadratic form $A=\begin{bmatrix}3&-1&1\\-1&5&-1\\1&-1&3\end{bmatrix}$ Characteristic equation of A is $\lambda^3-11\lambda^2+36\lambda-36=0$. Eigen values are $\lambda=2,3,6$. Corresponding eigen vectors are $\begin{bmatrix}1\\0\\-1\end{bmatrix},\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}1\\-2\\1\end{bmatrix}$. As the matrix A is symmetric and the eigen values are distinct, the eigen vectors are pairwise orthogonal and hence the normalized matrix $P=\begin{bmatrix}1/\sqrt{2}&1/\sqrt{3}&1/\sqrt{6}\\0&1/\sqrt{3}&-2/\sqrt{6}\\-1/\sqrt{2}&1/\sqrt{3}&1/\sqrt{6}\end{bmatrix}$ is orthogonal. Therefore $P^{-1}=P^T$. $P^TAP=D$ ……………………………………………….. Consider the transformation $X=PY$. Then $X^TAX=(PY)^TA(PY)=Y^T(P^TAP)Y=Y^TDY=2{y_1}^2+3{y_2}^2+6{y_3}^2$ is the required canonical form . Nature is positive definite.	1M 2M 1M 6M 1M 1M 1M 1M
6		Reduce the matrix $A=\begin{bmatrix}2&0&1\\0&2&0\\1&0&2\end{bmatrix}$ to the diagonal form and find A^4. **Sol**:Characteristic equation of A is $\lambda^3-6\lambda^2+5\lambda-6=0$. Eigen values are $\lambda=1,2,3$. Corresponding eigen vectors are $\begin{bmatrix}1\\0\\-1\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\1\end{bmatrix}$. $P^{-1}=\frac{1}{2}\begin{bmatrix}1&0&-1\\0&2&0\\1&0&1\end{bmatrix}$ $P^{-1}AP=D=\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}$ $A^n=PD^nP^{-1}\Rightarrow A^4=PD^4P^{-1}$	2M 2M 6M 1M 1M

		$=\begin{bmatrix} 41 & 0 & 40 \\ 0 & 16 & 0 \\ 40 & 0 & 41 \end{bmatrix}$	2M
7	a	Find the directional derivative of $f(x,y,z)=x^2-y^2+2z^2$ at $P(1,2,3)$ in the direction of the vector PQ, where Q is the point (5,0,4). **Sol**: Let $\phi : x^2-y^2+2z^2$ $\nabla\phi\|_P = 2\bar{i}-4\bar{j}+12\bar{k}$ $\overline{PQ}=\overline{OQ}-\overline{OP}=4\bar{i}-2\bar{j}+\bar{k}=\bar{a}$ Directional derivative $\frac{d\phi}{dr}=\nabla\phi\cdot\frac{\bar{a}}{\|\bar{a}\|}$ $=\frac{28}{\sqrt{21}}.$	 2M 1M 2M 2M
	b	Prove that $\nabla r^n = nr^{n-2}\bar{r}$ where $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}.$ **Sol**: $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$ and $r=\|\bar{r}\|=\sqrt{x^2+y^2+z^2}$ $\Rightarrow \frac{\partial r}{\partial x}=\frac{x}{r}, \frac{\partial r}{\partial y}=\frac{y}{r}, \frac{\partial r}{\partial z}=\frac{z}{r}$ Now $\nabla r^n=\sum \bar{i}\frac{\partial}{\partial x}r^n$ $=\sum \bar{i}nr^{n-1}\frac{\partial r}{\partial x}$ $=\sum \bar{i}nr^{n-1}\left(\frac{x}{r}\right)$ $=\sum \bar{i}nr^{n-2}x$ $=nr^{n-2}\sum x\bar{i}$ $=nr^{n-2}\bar{r}$	 2M 1M 2M 2M
8	a	Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2,-1,2). **Sol**: $\nabla\phi=2x\bar{i}+2y\bar{j}+2z\bar{k}$ & $\nabla\psi=2x\bar{i}+2y\bar{j}-\bar{k}$ At the point (2,-1,2), $\overline{n_1}=4\bar{i}-2\bar{j}+4\bar{k}$ & $\overline{n_2}=4\bar{i}-2\bar{j}-\bar{k}$ If θ is the angle between the normals then $\cos\theta=\frac{\overline{n_1}\cdot\overline{n_2}}{\|\overline{n_1}\|\|\overline{n_2}\|}$ $=\frac{8}{3\sqrt{21}}$	 2M 2M 2M 1M
	b	Show that $\nabla\cdot(f\bar{A})=f(\nabla.\bar{A})+\bar{A}\cdot(\nabla f)$ where f is the scalar function and $\bar{A}$ is the vector function. **Sol**: $\nabla\cdot(f\bar{A})=\sum\bar{i}\cdot\frac{\partial}{\partial x}(f\bar{A})$ $=\sum\bar{i}\cdot\left[\frac{\partial f}{\partial x}\bar{A}+f\frac{\partial\bar{A}}{\partial x}\right]$	 1M

		$=\sum\left[\bar{i}\cdot\frac{\partial f}{\partial x}\bar{A}+\bar{i}\cdot f\frac{\partial \bar{A}}{\partial x}\right]$ $=\sum\left[\bar{i}\frac{\partial f}{\partial x}\cdot\bar{A}+f\left(\bar{i}\cdot\frac{\partial \bar{A}}{\partial x}\right)\right]$ $=f\left(\nabla.\bar{A}\right)+\bar{A}\cdot(\nabla f)$	3M 3M
9	a	Find the work done in moving a particle in the force field $\bar{F}=3x^2\bar{i}+(2xz-y)\bar{j}+z\bar{k}$ along the straight line from (0,0,0) to (2,1,3). **Sol**: $\frac{x-0}{2-0}=\frac{y-0}{1-0}=\frac{z-0}{3-0}=t\Rightarrow x=2t,y=t,z=3t$ Therefore t varies from 0 to 1. Work done= $\int_C \bar{F}\cdot d\bar{r}$ $=\int_C 3x^2dx+(2xz-y)dy+zdz$ $=\int_{t=0}^{1}\left(24t^2+(12t^2-t)+9t\right)$ $=\left[24\frac{t^3}{3}+12\frac{t^3}{3}-\frac{t^2}{2}+9\frac{t^2}{2}\right]_0^1$ $=16$	2M 3M 2M
	b	Using Green's theorem, evaluate $\oint_C (xy+y^2)dx+x^2dy$ where C is bounded by $y=x$ and $y=x^2$. **Sol**: Region of integration, points of intersection,etc.,…………… Green's theorem states that $\oint_C \phi dx+\psi dy=\iint_R\left(\frac{\partial\psi}{\partial x}-\frac{\partial\phi}{\partial y}\right)dxdy$ $\frac{\partial\psi}{\partial x}-\frac{\partial\phi}{\partial y}=x-2y$ $\oint_C (xy+y^2)dx+x^2dy=\iint_R (x-2y)dxdy$ $=\int_{x=0}^{1}\int_{y=x^2}^{x}(x-2y)dxdy$ $=\int_{x=0}^{1}(x^4-x^3)dx$ $=-\frac{1}{20}$	1M 2M 2M 2M
10	a	Apply Gauss Divergence Theorem to find $\iint_S \bar{F}\cdot\bar{N}ds$ where $\bar{F}=4xz\bar{i}-y^2\bar{j}+yz\bar{k}$ taken over the cube bounded by x=0,x=1,y=0,y=1,z=0,z=1. **Sol**: Gauss Divergence Theorem states that $\int_S \bar{F}\cdot\bar{N}ds=\int_V div\bar{F}dv$ $div\bar{F}=4z-y$	2M 1M

		$\therefore \iint_S \bar{F}\cdot\bar{N}ds = \iiint_V (4z-y)dxdydz$ $= \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1}(4z-y)dxdydz$ $=3/2$	2M 2M
	b	Using Stokes theorem, evaluate $\int_C (x+y)dx+(2x-z)dy+(y+z)dz$, where C is the boundary of the triangle with vertices (2,0,0),(0,3,0),(0,0,6). **Sol**: Stoke's theorem states that $\int_C \bar{F}\cdot d\bar{r} = \int_S curl\bar{F}\cdot\bar{n}ds$ $curl\bar{F} = 2\bar{i}+\bar{k}$ Equation of the plane though the given points is $\frac{x}{2}+\frac{y}{3}+\frac{z}{6}=1 \Rightarrow 3x+2y+z=6$ Normal to this plane is given by $\nabla(3x+2y+z-6) = 3\bar{i}+2\bar{j}+\bar{k}$ Unit Normal= $\bar{n} = \frac{1}{\sqrt{14}}(3\bar{i}+2\bar{j}+\bar{k})$ Now $\int_C (x+y)dx+(2x-z)dy+(y+z)dz = \int_S curl\bar{F}\cdot\bar{n}ds$ $= \int_S \frac{1}{\sqrt{14}}(6+1)ds$ $= \frac{7}{\sqrt{14}}\int_S ds$ $= \frac{7}{\sqrt{14}}(3\sqrt{14})$ $=21$	2M 1M 1M 1M 2M

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